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The process $\gamma(^*) + p \to \eta_c + X$: a test for the perturbative QCD odderon

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Abstract. The rates of inclusive photo- and electroproduction of the η_c meson: $\gamma(^*) + p \to \eta_c + X$ are calculated in the triple Regge region, integrated over the diffractive mass X. For the Regge exchanges we use the hard pomeron and odderon, both being calculated in the framework of perturbative QCD. The integrated cross section depends upon the coupling of the BFKL pomeron to two $C = -1$ odderons, and it is found to be of the order of 60 pb for photoproduction and 1.5 pb at $Q^2 = 25\,\mathrm{GeV}^2$.

1 Introduction

The existence of the odderon [1], the partner of the pomeron which is odd under charge conjugation C, is an important prediction of perturbative QCD. It is a direct consequence of the number of colors N_c being greater than two. In the leading order, the odderon appears as a bound state of three reggeized gluons. Its experimental observation is a strong challenge for the experimentalists. A particular promising scattering process where the exchange of the odderon may be seen is the diffractive production of particles with a C-odd exchange, such as photo- and electroproduction of pseudoscalar mesons (PS), provided a large momentum scale is involved, which gives a justification for the use of perturbative QCD. This includes, in particular, the diffractive production of charmed pseudoscalar mesons, for example the process $\gamma + p \to \eta_c + p$. Correspondingly, a large amount of literature has been devoted to this class of diffractive processes. For large photon virtualities Q^2, for heavy mass PS mesons (such as η_c), and for large momentum transfers the relevant impact factors for the transition $\gamma(\gamma^*) \to$ PS have been calculated perturbatively [2]. As for the odderon structure, different models have been used: the exchange of three non-interacting gluons in a $C = -1$ state [2], and, more recently, the perturbative QCD odderon with intercept exactly unity [3, 4], used to calculate the production rates of the η_c in [5][1]. In both models, there is some uncertainty coming from the coupling of the odderon to the proton. Numerical estimates for the cross sections turn out to be somewhat different in these approaches. However, in all cases they are very small and, most unfortunately, do not grow with energy (in the case of the perturbative QCD odderon, they even slowly decrease with energy). This leaves little hope to see the odderon by increasing the energy of the reaction.

However, the situation may become different if, instead of the quasielastic process $\gamma + p \to \eta_c + p$, one considers the inclusive cross section $\gamma + p \to \eta_c + X$ in the triple Regge region. In this case the odderon does not couple directly to the quarks of the target proton but rather to the diffractive system "X" which, for high masses, can be modelled by a cut gluon ladder, the gluon density inside the proton. The proton is therefore coupled to the cut gluon ladder, i.e. the pomeron, and this coupling is known through the gluon density. This fact permits one to avoid the previously mentioned uncertainties in the odderon–proton coupling. Together with this process also the low mass diffractive state (the proton) with the meaning of a double odderon exchange is usually considered.

In the Regge language this new situation basically involves the coupling of two odderons to a cut pomeron, the POO vertex. Since we are using perturbative QCD both for the odderon and for the cut pomeron, also this vertex has to be calculated in perturbative QCD. This has been done in [8]: the vertex has been obtained in an analysis of a six-gluon amplitude D_6. In our application of this vertex we shall restrict ourselves to the leading large-N_c limit, which leads to a relatively simple form of D_6. In [8] it was also shown that the full amplitude D_6 can be decomposed into the sum of two contributions, where the first one results from the reggeization of the gluon, and the second one contains the POO vertex. Correspondingly, also our cross section comes as the sum of two pieces (denoted by

[1] A different approach, a non-perturbative odderon based upon the idea of a "stochastic QCD vacuum" has been used in [6].

P and POO, respectively). The second one corresponds to the normal "triple Regge picture" where the pomeron splits into two odderons, whereas the first one is related to reggeization of the gluon and leads the exchange of three non-interaction gluons in the odderon channel. We will calculate both. For the odderon states we will use the solution found in [3], which has a maximal intercept (unity) and a very simple analytical form. This solution has already been used by us to calculate the odderon exchange in the process $\gamma + p \to \eta_c + p$ [5].

The use of these elements allows one to compute the diffractive cross section $\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\mathrm{d}M^2}$ for the process $\gamma + p \to \eta_c + X$. Our perturbative QCD analysis only depends upon one free parameter, the coupling of the gluon ladder to the proton: this coupling will be fixed by fitting the model to the gluon density of the proton. To simplify the calculations we restrict ourselves to the integrated (over M^2) cross section: the calculation of the differential (in M^2) cross section requires a slightly different treatment of the D_6 amplitude which will not be pursued in the present work. Nevertheless the most inportant and basic information is given by the integrated cross section.

As we have indicated before, we expect that the cross section for the inclusive process $\gamma + p \to \eta_c + X$ is larger than that for the quasielastic process $\gamma + p \to \eta_c + p$. We know that the cut gluon ladder grows as $\exp \Delta y$, where y is the rapidity gap between the proton and the POO vertex, and Δ is the value of the pomeron intercept minus unity. From this it follows that the bulk of the inclusive cross section will come from the region where y is as large as possible. i.e. close to the total rapidity of the process (note however, that, in order to see the exchange of an odderon, one needs also a large rapidity gap between the outgoing η_c and the diffractive system). In other words, the mass of the diffractive system "X" wants to become as large as possible. Because of this growth (with energy) of the cut gluon ladder we expect to see a strong enhancement of the inclusive cross section at high energies, compared to the quasielastic process $\gamma + p \to \eta_c + p$ where the gluon ladder is absent. The comparison of our results with the quasielastic cross section is made difficult by the intrinsic uncertainty of the odderon–proton coupling: a recent analysis shows that the estimate obtained in [2] and also adopted in [5] may have used a too large value of this coupling and has to be reduced: if this is the case, the cross section obtained in the present paper is, in fact, much larger that the quasielastic one.

Our paper is organized as follows. In the next section we shall briefly recall some results which constitute our starting point to attack the problem, the cross section formula. In Sect. 3 the first contribution (P) is considered, and the corresponding integrated (in M^2) cross section is written in terms of a multidimensional integral which, later on, will be computed numerically. In Sects. 4 and 5 the structure of the second contribution (POO) is considered. Both contributions are calculated in the large N_c limit, which leads to considerable simplifications and shows a symmetry shared by the leading odderon states [3]. Finally, the numerical analysis is presented and discussed in Sect. 6, followed by the conclusions in Sect. 7.

2 The cross section formula in perturbative QCD

We start from the analysis [8] of QCD Feynman diagrams in the leading log s approximation, and we recapitulate the main results. The approach taken from [8] is a generalization of a previous analysis [9] of the four-gluon system (related to the triple pomeron vertex): it extends this analysis up to six gluons in the t-channel, and so it encounters, for the first time, the two-odderon state. As described in [8], the analysis of Feynman diagrams in the high energy limit leads to a tower of gluon amplitudes, D_2, D_3, D_4, D_5, and D_6, which satisfy a set of coupled integral equations. These functions are non-amputated, i.e. they contain reggeon denominators for the outgoing (reggeized) gluon states [9]. The latter are more convenient degrees of freedom than the elementary gluons in this kinematics. In the present context we are interested in the D_6 amplitude which can be used to build the cross section, integrated in the diffractive mass.

In the analysis in [8,9], all functions $D_2, \dots$ start from the impact factor of a virtual photon which splits into a quark–antiquark pair. In the present case, the external particle is the proton: assuming that, to a good approximation, the proton can be viewed as a quark–diquark system, the coupling of the gluons to the proton should have the same structure as in the photon case; only the overall normalization of this coupling has to be treated as a phenomenological parameter.

The diagrammatic structure of the differential cross section $\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\mathrm{d}M^2}$ for the process $\gamma + p \to \eta_c + X$ is illustrated in Fig. 1a. In the two exchange channels one recognizes the two-odderon states, consisting of three gluons with pairwise interactions. The structure of the blob (related to D_6) will be discussed in a future paper. When the integration over the squared missing mass M^2 is performed, the expression for the cross section $\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \int \mathrm{d}M^2 \frac{\mathrm{d}^2\sigma}{\mathrm{d}t\mathrm{d}M^2}$ simplifies. The result is illustrated in Fig. 1b: the blob now stands for D_6 which can directly be taken from [8]. It depends upon the angular momentum variable ω which is conjugate to the total rapidity Y.

The cross section can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \xi \sum_{i=1,2} \int \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \mathrm{e}^{Y\omega} \int \frac{\mathrm{d}\mu_1 \mathrm{d}\mu_2}{\prod_{i=1}^{6} k_i^2} \times \Phi^i(1,2,3)[\Phi^i(4,5,6)]^* D_6(1,2,3,4,5,6;\omega)\,. \quad (1)$$

Here Φ^i denotes the impact factor for the transitions $\gamma^* \to \eta_c$ with the photon polarizations $i = 1,2$ and the color structure d_{abc}. Y is the overall rapidity; $t = -q^2$ is the invariant associated to the momentum transfer across the impact factor, and the arguments $1,2,3$ and $4,5,6$ refer to both the color indices a_i and transverse momenta, k_i, of the gluons exchanged in the initial amplitude $i = 1,2,3$ and final (conjugated) amplitude $i = 4,5,6$. Finally, $\mathrm{d}\mu_1$ is the integration measure for the three t-channel gluons on the LHS:

$$\mathrm{d}\mu_1 = \mathrm{d}^2k_1 \mathrm{d}^2k_2 \mathrm{d}^2k_3 \delta^2(k_1 + k_2 + k_3 - q) \quad (2)$$

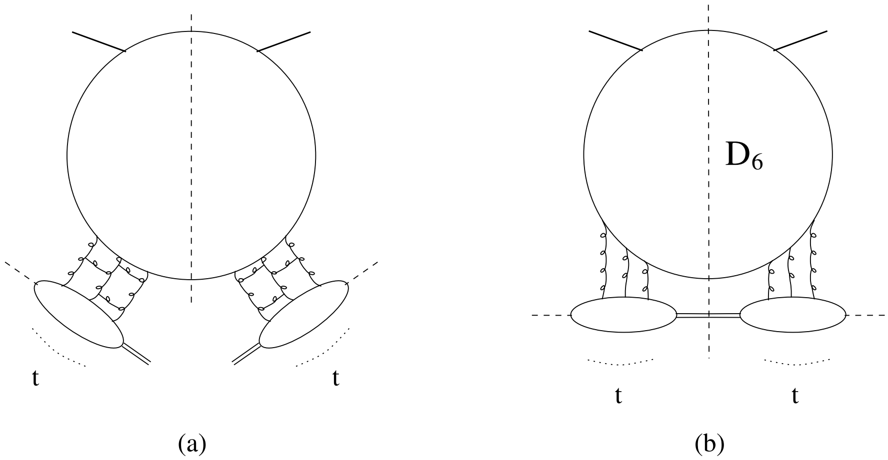

Fig. 1. Illustration of the process $\gamma^* + p \to \eta_c + X$. Diffractive cross section differential **a** and integrated **b** in the diffractive mass

and $d\mu_2$ is the analogous integration measure for the three t-channel gluons 4, 5, 6 on the RHS. The normalization factor ξ will be discussed in the next section.

From the analysis [8] of the coupled integral equations it follows that D_6 can be presented as a sum of different terms. One term (denoted by D_6^{R}) is obtained by collecting the reggeizing pieces: the outgoing six-gluon state may contain configurations where a pair of two gluons is in an antisymmetric color octet configuration, which satisfies the BFKL bootstrap condition and collapses into a single gluon. As a result, one obtains contributions with a smaller number of reggeized gluons. It is convenient to separate these configurations from the rest, i.e. to define the sub-amplitude D_6^{I} which is "irreducible" with respect to this reduction procedure. This reduction leads to the decomposition $D_6 = D_6^{\mathrm{R}} + D_6^{\mathrm{I}}$, separating the reggeizing (R) and irreducible (I) parts. The D_6^{I} term ((6.3) of [8]) is rather lengthy; however, for an odderon in the (123) and (456) channels, we will need, in the large-N_c limit, only one term, denoted by W, which describes the transition of two reggeized gluons into six reggeized gluons: all other terms will be shown (see Sect. 4) to be suppressed by a factor of $1/N_c^2$ (or even higher powers of this). It is this piece of D_6^{I} which yields the POO vertex.

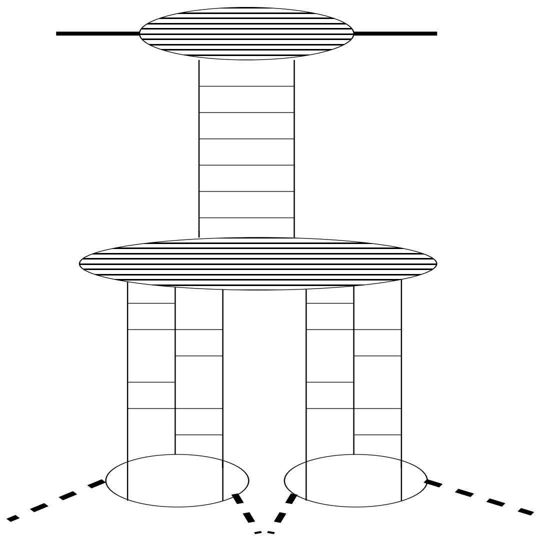

Fig. 2. Illustration of the process $\gamma^* + p \to \eta_c + X$: the blob in the center of the figure denotes the transition vertex pomeron $\to$ 2 odderons

Diagrammatically, the piece of D_6^{I} which, in the large-N_c limit contains the pomeron $\to$ odderon vertex has the structure shown in Fig. 2. The internal blob – with two gluons entering from above and six gluons leaving below – defines the pomeron $\to$ odderon (POO) vertex, and its color structure is quite simple:

$$\delta_{b,b'} d_{a_1 a_2 a_3} d_{a_4 a_5 a_6} W(1,2,3|4,5,6)\,, \tag{3}$$

where the b, b' are the color labels of the reggeized gluons of the ladder above the POO vertex, a_i the color indices of the reggeized gluons below the vertex (counting from left to right). The arguments of the function W refer to the momenta of the gluons. Below the POO vertex, we have the two non-interacting odderons: the pairwise interactions inside (123) and (456) lead to the color singlet odderon Green functions. We note that this simple form emerges only after taking the large-N_c limit. In the more general case of finite N_c, the expression (3) has to be summed over permutations of the indices (123456). Moreover, in Fig. 2 below the POO vertex, we would have to include *all* pairwise interactions between the reggeized gluons. It is only in the large-N_c limit that any rung which connects the two color singlet (123) and (456) costs a suppression factor of the order $1/N_c^2$ and, therefore, can be neglected.

The D_6^{R} term is nothing but a sum of BFKL ladders in which, at the lower end, the reggeized gluons split into two three or four elementary gluons. Inserting this sum into the blob in Fig. 1b and taking the large-N_c limit, we arrive at structures illustrated in Fig. 3: the BFKL ladder couples to odderon states consisting of three non-interacting gluons. Below we will discuss this in further detail: starting from the color structure of D_6^{R}, given in [8], (6.2), it can be shown that all these contributions are subleading in $1/N_c$; in our further

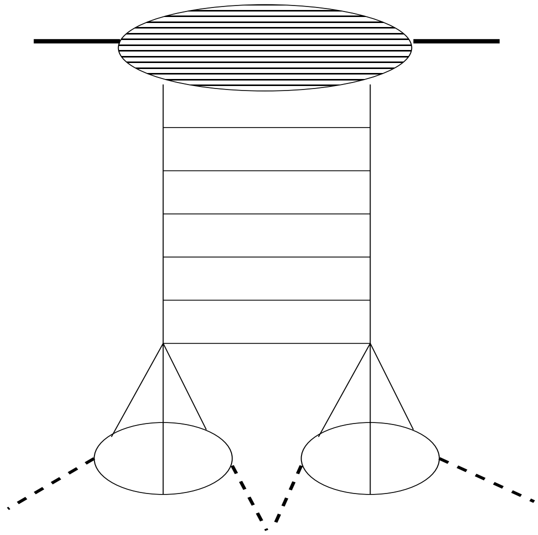

Fig. 3. The second contribution to the same process as Fig. 1: the pomeron couples to the two odderons which consist of three non-interacting gluons

discussion we will keep one of them which is suppressed by a factor $1/N_c$ (all others are further suppressed).

All this discussion refers to Fig. 1b which illustrates the *integrated* inclusive cross section. In order to derive the pQCD formula for the *differential* cross section, an alternative separation of D_6 is more suitable. For the case of D_4 which leads to the triple pomeron vertex, such a separation has been discussed in [9,10]: as a result, a slightly different expression for the triple pomeron emerges. This question will be discussed in a forthcoming paper.

3 Contribution of the reggeizing piece D_6^{R}

Let us now analyze the two contribution to our inclusive cross section in some detail. We start with the ideologically (but not calculationally!) simpler part of the transition rate corresponding to the reggeizing piece, D_6^{R}, of [8] (Fig. 3). In the following, we will denote this piece by the superscript P. The corresponding inclusive cross section is given by the expression

$$\frac{\mathrm{d}\sigma^{(P)}}{\mathrm{d}t} = \xi \sum_{i=1,2} \int \frac{\mathrm{d}\omega}{2\pi \mathrm{i}} \mathrm{e}^{Y\omega} \int \frac{\mathrm{d}\mu_1 \mathrm{d}\mu_2}{\prod_{i=1}^{6} k_i^2} \times \Phi^i(1,2,3)[\Phi^i(4,5,6)]^* D_6^{\mathrm{R}}(1,2,3,4,5,6;\omega)\,. \quad (4)$$

The function D_6^{R} depending on all gluonic momenta is the reggeizing piece of the six-gluon amplitude found in [8]. It is given by the sum of BFKL pomerons depending on various partial sums of the momenta of the six gluons $1,\ldots,6$, multiplied by certain color factors. All color factors are obtained by permutations of inital (123) or final (456) gluons from

$$d^{a_1a_2a_3a_4a_5a_6} = \mathrm{tr}(t^{a_1}t^{a_2}t^{a_3}t^{a_4}t^{a_5}t^{a_6}) + \mathrm{tr}(t^{a_6}t^{a_5}t^{a_4}t^{a_3}t^{a_2}t^{a_1})\,, \quad (5)$$

where t^a is the quark color matrix. This evidently gives eight different color factors, so that D^{R} contains eight terms with different color structure. Coupling D^{R} to the two impact factors in (1) one has to contract its color indices with a product $d_{a_1a_2a_3}d_{a_4a_5a_6}$ corresponding to the $C=-1$ exchange of three gluons. Then one finds that all color factors in D^{R} are transformed into the same common color factor $F_c^{(\mathrm{R})}$

$$d^{a_1a_2a_3a_4a_5a_6}d_{a_1a_2a_3}d_{a_4a_5a_6} = \frac{(N_c^2-1)^2(N_c^2-4)^2}{8N_c^3} \equiv N_c^5 F_c^{(P)}\,. \quad (6)$$

At large N_c it is $\sim N_c^5/8$, in correspondence with the general rules of the $1/N_c$ expansion. Note however that at $N_c = 3$ its value $200/27 \sim 7.4$ is nearly 4 times smaller than given by the $N_c \to \infty$ limit $243/8 \sim 30$. Finally the overall factor ξ is equal to

$$\xi = \frac{1}{2}\frac{\pi}{16\pi^3}\left(\frac{1}{4\cdot 4(2\pi)^6(3!)}\right)^2\,. \quad (7)$$

The first factor $1/2$ corresponds to the averaging over the two photon polarizations (we just consider transverse photon). Then one has the standard $1/(16\pi^3)$ phase space volume for the diffractive process times a π factor due to angular averaging. In the squared term there is the contribution of $(2\pi)^{-4}$ absent in each $\mathrm{d}\mu$ and an extra $(2\pi)^{-2}$ which we associate to the impact factor [2] in our normalization. Moreover one has the symmetry factor $1/3!$ for each of the gluon triplets, $1/4$ from the color factor which is in fact $(1/4)d_{abc}$ and $1/4$ from the integrations over s_i, instead of over k_{i-} in the definition of the impact factor.

Separating the common factor $g_s^4 N_c^5 F_c^{(P)}$, where g_s is the strong coupling constant, we shall reproduce the rest of the amplitude D^{R} from [8] in a simplified manner, taking into account that, first, in the pomeron of Fig. 3 the total momentum of the two gluons is zero and, second, in the high-energy limit the amplitude for the leading contribution is symmetric in the two gluons. Thus this ampitude $P(k)$ (amputated, that is, without external gluon propagators) depends only on one of the gluon momenta. In terms of $P(k)$ the reggeizing piece is then given by a sum of 31 terms:

$$D_6^{\mathrm{R}} = \sum_{i=1}^{6} P(i) - \sum_{i\neq k=1}^{3} P(ik) - \sum_{i\neq k=4}^{6} P(ik) - \sum_{i=1}^{3}\sum_{k=4}^{6} P(ik) + \sum_{i\neq k=1}^{3}\sum_{l=4}^{6} P(ikl) + P(q)\,. \quad (8)$$

Here the notations il and ilm denote sums of gluon momenta $k_i + k_l$ and $k_i + k_l + k_m$ respectively. Expression (8) can further be simplified if we take into account that in (4) the integration over all momenta is done for a function which is totally symmetric in the gluons (123) and (456). Moreover, it is symmetric under the interchange $(123) \leftrightarrow (456)$. Therefore, on the RHS of (8) the terms inside each sum are identical, and we get

$$D_6^{\mathrm{R}} = 6P(1) - 6P(12) - 9P(14) + 9P(124) + P(q)\,. \quad (9)$$

The $\gamma^* \to \eta_c$ impact factor is given by [2]

$$\Phi^i(1,2,3) = b\epsilon_{ij}\frac{q_j}{q^2}\phi(1,2,3), \quad i,j=1,2. \tag{10}$$

Here

$$\phi(1,2,3) = \sum_{i=1}^{3}\frac{q(q-2k_i)}{M^2+(q-2k_i)^2} - \frac{q^2}{M^2+q^2} \tag{11}$$

and $M^2 = Q^2 + 4m_c^2$, where Q^2 is the photon virtuality and m_c the charmed quark mass. The coefficient b is given by

$$b = \frac{16}{\pi}e_c g_s^3 \frac{1}{2} m_{\eta_c} b_0\,, \tag{12}$$

where $e_c = (2/3)e$ is the electric charge of the charmed quark m_{η_c} is the η_c meson mass and b_0 can be determined from the known radiative width $\Gamma(\eta_c \to \gamma\gamma)= 7\,\text{keV}$:

$$b_0 = \frac{16\pi^3}{3e_c^2}\sqrt{\frac{\pi\Gamma}{m_{\eta_c}}}\,. \tag{13}$$

The impact factor (10) is symmetric in the three gluon momenta, and it vanishes if any of the momenta goes to zero. Taking in (4) the product of the two impact factors and summing over polarizations we obtain

$$F_{\gamma^*\to\eta_c}\phi(1,2,3)\phi(4,5,6)\,, \tag{14}$$

where

$$F_{\gamma^*\to\eta_c} = \frac{b^2}{q^2}\,. \tag{15}$$

The pomeron $P(k)$ can be presented as a convolution of the BFKL Green function with the color distribution $\rho(r)$ in the hadronic target:

$$P(k) = -g_s^2\int \mathrm{d}^2r' G(Y,k,r')\rho(r'). \tag{16}$$

Note the minus sign. After transforming the initial momentum space expression for the impact factor into the coordinate space, the impact factor is proportional to $1-\exp(\mathrm{i}kr)$. When multiplying with the pomeron Green function and doing the k-integral, there is no contribution from the "1" (since, in coordinate space, the pomeron Green function vanishes when both arguments coincide), and the non-zero contribution comes from the second term, $-\exp(\mathrm{i}kr)$. The Green function has to be taken in a mixed representation, momentum k at the odderon side, coordinate r' at the proton side. Also, one side of the Green function is amputated, the other not. We have

$$G(Y,k,r') = -\frac{1}{8\pi^2}qr'\int\frac{\mathrm{d}\nu}{\nu^2+1/4}\mathrm{e}^{Y\omega(\nu,0)}\left(\frac{qr'}{2}\right)^{2\mathrm{i}\nu} \tag{17}$$

with

$$\omega(\nu,n) = 2\bar\alpha_s\left(\psi(1) - \mathrm{Re}\,\psi\left(\frac{1+|n|}{2}+\mathrm{i}\nu\right)\right),$$

$$\bar\alpha_s = \frac{\alpha_s N_c}{\pi}. \tag{18}$$

At large rapidities small ν's dominate. Due to the finite dimension of the target, in (17) the values of r' are limited by the radius R. So we can neglect factor the $(r')^{2\mathrm{i}\nu}$, and the integration over r' will be replaced by the average transverse dimension of the target R. So at high Y the Green function gives a factor

$$-\frac{1}{2\pi^2}kR\sqrt{\frac{\pi}{aY}}\mathrm{e}^{\Delta Y}\exp\left(-\frac{\ln^2 kR}{4aY}\right). \tag{19}$$

Here $\Delta = \omega(0,0) = \bar\alpha_s\,4\ln 2$ is the BFKL intercept with $a = 7\bar\alpha_s\zeta(3)$. The second exponential factor cuts the integration over k to values $\ln^2 k < aY$. However, since the integration in (4) is in fact convergent, we may drop this factor at high enough Y. Putting (19) into (8) we get for the cross section

$$\frac{\mathrm{d}\sigma^{(P)}}{\mathrm{d}t} = \xi g_s^6\frac{b^2}{2\pi^2q^2M^3}N_c^5F_c^{(P)}R\mathrm{e}^{\Delta Y}\sqrt{\frac{\pi}{aY}}I\left(\frac{q}{M}\right), \tag{20}$$

where the dimensionless function $I(q/M)$ is given by the integral

$$\begin{aligned} I\left(\frac{q}{M}\right) &= M^3\int\frac{\mathrm{d}\mu_1\mathrm{d}\mu_2}{\prod_{i=1}^6 k_i^2} \\ &\times \phi(1,2,3)\phi(4,5,6)\left(6|k_1| - 6|q-k_1| - 9|k_1-k_4|\right. \\ &\left.+9|q-k_1-k_4| + q\right). \end{aligned} \tag{21}$$

The cross section (20) is of the order $\alpha_s(\alpha_s N_c)^5$. The 8-dimensional integral (21) is non-factorizable and can be done only numerically.

As an alternative way of evaluating this contribution to the cross section, one might try to factorize the integration over gluonic momenta and to transform the pomeron amplitude to the coordinate space using

$$k^{1+2\mathrm{i}\nu} = -(1+4\nu^2)\int\frac{\mathrm{d}^2r}{2\pi r^3}\left(\frac{2}{r}\right)^{2\mathrm{i}\nu}\mathrm{e}^{\mathrm{i}kr}\,. \tag{22}$$

Taking the limit $\nu \to 0$ and introducing the function of r

$$h(r) = \int\frac{\mathrm{d}\mu_1\phi(1,2,3)}{k_1^2k_2^2k_3^2}\mathrm{e}^{\mathrm{i}k_1r}\,, \tag{23}$$

we find the integral I as

$$\begin{aligned} I\left(\frac{q}{M}\right) &= -M^3\int\frac{\mathrm{d}^2r}{2\pi r^3} \\ &\Big\{\Big(6h(r)h(0) - 6\mathrm{e}^{\mathrm{i}qr}h(-r)h(0) - 9h(r)h(-r) \\ &+ 9\mathrm{e}^{\mathrm{i}qr}h^2(-r) + \mathrm{e}^{\mathrm{i}qr}h^2(0)\Big) - \Big(r=0\Big)\Big\}. \end{aligned} \tag{24}$$

In obtaining this expression we used the fact that the change $q \to -q$ is equivalent to changing $r \to -r$ in $h(r)$. Also, we have taken into account that $P(k=0) = 0$, in order to subtract the value of the brackets at $r = 0$ and thus to improve the convergence of the integral at $r = 0$. Passing to the function

$$h_1(r) = h(r) - \mathrm{e}^{\mathrm{i}qr} h(-r), \quad h_1(0) = 0, \tag{25}$$

we can rewrite (23) in a simpler form, which also shows that the right-hand side is real:

$$I\left(\frac{q}{M}\right) = -M^3 \int \frac{\mathrm{d}^2 r}{2\pi r^3} \tag{26}$$
$$\left(6h(0)\mathrm{Re}\, h_1(r) - \frac{9}{2}|h_1(r)|^2 - (1-\cos(qr))h^2(0)\right).$$

Now we have only six integrations to be done numerically. However the presence of the oscillating factor makes such calculations very difficult. We therefore use (21) and performed the integrations by Monte Carlo methods. The results will be presented in Sect. 6, together with the contribution from the POO vertex.

4 The POO vertex in leading order in N_c

Next let us consider the irreducible part D_I of the amplitude for six reggeized gluons. Its contribution to the cross section will be denoted by the superscript POO. Starting from [8] (see (6.3)), we note that the RHS satisfies a BFKL-like equation which we write in the symbolic form

$$(H_6 - E)D_I = D_I^{(0)}\,. \tag{27}$$

Here H_6 is the total Hamiltonian for six reggeized gluons, which is a sum of pairwise interactions and of gluon trajectories and describes their evolution, without changing the number of gluons. The energy $E = 1 - j = -\omega$ is just one minus the intercept. The driving term of the equation is a sum of terms which describe transitions with a change of the number of gluons, from "irreducible" configurations of two, four, or five gluons to six gluons. At this moment it is important to invoke the approximation of large number of colors $N_c \to \infty$. In this approximation, any interaction inside the outgoing six-gluon state which connects colorless groups of gluons is damped by $1/N_c^2$ and can be neglected. This means, in particular, that once a pair of states with color color structure of two odderons is formed in the driving term, H_6 in (27) contains no further interaction between these two-odderon states. All what H_6 does is to build up the bound states of the gluons (123) and (456). So in order to find the terms relevant for the POO transitions we only have to see whether the final two-odderon states couples to the driving term. One immediately sees that, at large N_c, the irreducible configurations of four and five gluons, D_4^{I} and D_5^{I}, reduce to the splitting of the initial pomeron into two pomerons and thus cannot couple to the two-odderon final state. Therefore, the transitions of interest can only occur in terms which describe transitions of two gluons to six gluons. In [8] four such terms of different color structure were found.

The first group is given by a sum

$$d_{a_1a_2a_3}d_{a_4a_5a_6}W(1,2,3|4,5,6)$$
$$+\, d_{a_1a_2a_4}d_{a_3a_5a_6}W(1,2,4|3,5,6) + \ldots, \tag{28}$$

where the sum extends over all (ten) partitions of the six gluons into two groups containing three gluons each. Projecting onto the two-odderon color state we find the color factor for the first term in (28)

$$d_{a_1a_2a_3}d_{a_4a_5a_6}d_{a_1a_2a_3}d_{a_4a_5a_6} = \frac{(N_c^2-1)^2(N_c^2-4)^2}{N_c^2}$$
$$\equiv N_c^6 F_c^{(POO)} \sim N_c^6, \tag{29}$$

whereas for all the rest terms we have

$$d_{a_1a_2a_4}d_{a_3a_5a_6}d_{a_1a_2a_3}d_{a_4a_5a_6} = \frac{(N_c^2-1)(N_c^2-4)^2}{N_c^2}$$
$$\sim N_c^4. \tag{30}$$

So at large N_c we will retain only the first term in the sum, (28).

Apart from the W terms, the remaining driving terms in (6.3) of (27) with transitions from two to six gluons contain three more groups of terms of different color structure. Terms denoted by L in [8] are given by a sum

$$f_{a_1a_2a_3}f_{a_4a_5a_6}L(1,2,3|4,5,6)$$
$$+\, f_{a_1a_2a_4}f_{a_3a_5a_6}L(1,2,4|3,5,6) + \ldots, \tag{31}$$

with the sum, again, extending over all (ten) partitions of the six gluons into two groups containing three gluons each, and the function being described in [8]. Obviously these terms give zero when projected onto the color state of two odderons.

Finally, the terms denoted by I and J are given by sums:

$$d^{a_1a_2a_3a_4}\delta_{a_5a_6}I(1,2,3,4|5,6)$$
$$+\, d^{a_1a_2a_3a_5}\delta_{a_4a_6}I(1,2,3,5|4,6) + \ldots \tag{32}$$

and

$$d^{a_2a_1a_3a_4}\delta_{a_5a_6}I(1,2,3,4|5,6)$$
$$+\, d^{a_2a_1a_3a_5}\delta_{a_4a_6}I(1,2,3,5|4,6) + \ldots, \tag{33}$$

with the sum extending over all partitions of six gluons into two groups with four and two gluons, known function I and J and

$$d^{a_1a_2a_3a_4} = \mathrm{tr}(t^{a_1}t^{a_2}t^{a_3}t^{a_4}) + \mathrm{tr}(t^{a_4}t^{a_3}t^{a_2}t^{a_1}). \tag{34}$$

Projecting onto the color state of two odderons we find non-zero color factors of the type

$$d^{a_1a_2a_4a_5}\delta_{a_3a_6}d_{a_1a_2a_3}d_{a_4a_5a_6} = \frac{(N_c^2-1)(N_c^2-4)^2}{4N_c^2} \sim N_c^4\,. \tag{35}$$

So although these terms seem to involve POO transitions, they are down by a factor N_c^2 as compared to (29).

So in the end we find that, in the large N_c limit, the transitions POO are fully described by the function $W(1,2,3|4,5,6)$ which represents a convolution of the pomeron with a POO vertex. The functional form of W in [8] is rather complicated, however closer inspection shows a surprisingly simple structure [3]. Let us briefly recapitulate this structure. It is convenient to introduce two operators which transform a function of the momenta of two gluons into a new function which depends upon the momenta of three gluons. Namely, define $\hat{S}$ to be an operator acting on two-gluon states and with values on the three-gluon states, which performs an antisymmetrization in the two incoming gluons, splits the first of them in two and sums over the cyclic permutations of the outgoing gluons:

$$\hat{S}(1,2,3|1',2')\phi(1',2') = \frac{1}{2}\sum_{(123)}[\phi(12,3)-\phi(23,1)]\,. \tag{36}$$

Next define another operator $\hat{P}$ which performs an antisymmetrization in the two incoming gluons and splits the first of them in three outgoing gluons:

$$\hat{P}(1,2,3|1',2'))\phi(1',2') = \frac{1}{2}[\phi(123,0)) - \phi(0,123)]\,. \tag{37}$$

Apart from these operators we introduce a function $f(1,2|3,4)$, antisymmetric in the first and second pairs of gluons and symmetric under the interchange $(12) \leftrightarrow (34)$, as a sum of the functions $G(1,2,3)$, which were introduced in [9,10] in the context of the three-pomeron vertex:

$$f(1,2|3,4) = G(1,23,4) - G(2,13,4) - G(1,24,3) + G(2,14,3)\,. \tag{38}$$

The explicit form of the general function $G(1,2,3)$ is not important for our purposes (it can be found e.g. in [9,10]). We only have to know that

$$G(1,2,3) = G(3,2,1), \quad G(0,2,3) = G(1,2,0) = 0\,, \tag{39}$$

and that, up to a coefficient, $G(1,0,3)$ is given by the BFKL Hamiltonian H_2 applied to the pomeron:

$$G(1,0,3) = -\frac{1}{N_c}\Big(H_2P\Big)(1,3)\,. \tag{40}$$

In terms of $\hat{S}$, $\hat{P}$ and f we find

$$W(1,2,3|4,5,6) = -\frac{1}{8}g_s^4(\hat{S}_1-\hat{P}_1)f_{12}(\hat{S}_2^\dagger-\hat{P}_2^\dagger)\,, \tag{41}$$

where the indices 1 and 2 refer to the triplets of gluons (123) and (456), respectively.

5 Part of the cross section with a POO transition

To find the cross section corresponding to the η_c production via the POO transition (Fig. 1) we have to couple the POO vertex with the two odderons attached to the initial and final $\gamma^* \to \eta_c$ impact factors. To write it in a compact form we introduce the Green functions $G_3^{(1)}$ and $G_3^{(2)}$ for the initial and final odderons composed of gluons 123 and 456, respectively. They evolve the odderon state in the rapidity interval with length $y < Y$. Then using (41) we can write the cross section as

$$\frac{\mathrm{d}\sigma^{(POO)}}{\mathrm{d}t} = -\frac{1}{8}g_s^4N_c^6F_c^{(POO)}\xi\sum_{i=1,2}\int\mathrm{d}y \times \langle\Phi_1^i|G_3^{(1)}(\hat{S}_1-\hat{P}_1)f_{12}(\hat{S}_2^\dagger-\hat{P}_2^\dagger)G_3^{(2)}|\Phi_2^i\rangle\,, \tag{42}$$

where it is assumed that the averaging is done independently over the gluons 123 (index 1) and 456 (index 2), the function f depending on both groups of variables.

At this point we recall that the full set of odderon states and consequently the full Green function G_3 are unknown. We are going to use a part of it corresponding to the solutions found in [3], which have the maximal intercept of all known states and besides this have a non-zero coupling to the perturbative $\gamma^* \to \eta_c$ impact factor. These odderon states are expressed via the known antisymmetric pomeron states $E^{(\nu,n)}$ with odd values of n:

$$\Psi^{(\nu,n)}(1,2,3) = c(\nu,n)\frac{1}{k_1^2k_2^2k_3^2}\hat{S}(1,2,3|1',2'){k'_1}^2{k'_2}^2E^{(\nu,n)}(1',2')\,, \tag{43}$$

where

$$c(\nu,n) = \sqrt{\frac{g_s^2N_c}{-3(2\pi)^3\omega(\nu,n)}} \tag{44}$$

and $\omega(\nu,n)$ is given by (18). The part of the Green function G_3 corresponding to these states then aquires a form similar to the pomeron Green function [5]

$$G_3(y_1|1,2,3|1',2',3') = \sum_{\text{odd }n}\int\mathrm{d}\nu \times \mathrm{e}^{y_1\,\omega(\nu,n)}\beta(\nu,n)\Psi^{(\nu,n)}(1,2,3){\Psi^{(\nu,n)}}^*(1',2',3')\,, \tag{45}$$

with

$$\beta(\nu,n) = \frac{(2\pi)^2(\nu^2+n^2/4)}{[\nu^2+(n-1)^2/4][\nu^2+(n+1)^2/4]} \tag{46}$$

and $\Psi^{(\nu,n)}$ given by (43).

Now we use the fact that we only study our cross section in the region where both the rapidity of the pomeron y and that of the odderon $y_1 = Y - y$ are large. This allows one to retain in (45) only the branch $|n| = 1$ with a maximal intercept and also restrict the integration over ν to small values. In the limit $\nu \to 0$ the coupling of the odderon to the $\gamma^* \to \eta_c$ impact factor was calculated in [5] to be

$$\langle \Phi^i_{\gamma\to\eta_c} | \Psi^{(\nu,n)} \rangle = -\frac{\mathrm{i}}{\pi} b\epsilon_{ij} \frac{q_j}{q} \frac{1}{c(\nu,n)} q^{2\mathrm{i}\nu} \frac{1}{q^2+M^2} . \quad (47)$$

After summation over the photon polarizations the two matrix elements (47) provide a factor

$$\frac{1}{\pi^2} \frac{b^2}{c(\nu_1,n_1)c(\nu_2,n_2)} \frac{1}{(q^2+M^2)^2} q^{2\mathrm{i}(\nu_1-\nu_2)} , \quad (48)$$

where (ν_1, n_1) and (ν_2, n_2) refer to the summation and integration variables in $G_3^{(1)}$ and $G_3^{(2)}$ respectively. At finite values of q one can neglect the last factor in (48).

We are left with the matrix element

$$T \equiv \langle \Psi_1^{(\nu_1 n_1)} | (\hat{S}_1 - \hat{P}_1) f_{12} (\hat{S}_2^\dagger - \hat{P}_2^\dagger) | \Phi_2^{(\nu_2,n_2)} \rangle . \quad (49)$$

Its calculation is greatly simplified by the property of the odderon state (43) found in [3]. For any function $\phi(1,2)$ of two gluon momenta and odderon state (43) one has

$$\langle \Psi^{(\nu,n)} | (\hat{S} - \hat{P}) | \phi \rangle = \frac{1}{c(\nu,n)} \langle E^{(\nu,n)} | \phi \rangle . \quad (50)$$

Note that the matrix element on the left-hand side is taken in the space of three gluons, whereas that on the right-hand side is taken in the space of only two gluon momenta. This property greatly simplifies the matrix element (49). Using (50) we get for it

$$T = \frac{1}{c(\nu_1,n_1)c(\nu_2,n_2)} \langle E_1^{(\nu_1,n_1)} | f_{12} | E_2^{(\nu_2,n_2)} \rangle . \quad (51)$$

Now we again use the fact that y_1 is large and so only values $|n| = 1$ and $|\nu| << 1$ contribute. At $|n| = 1$ and small ν the pomeron wave funtions entering (51) reduce to δ functions of gluon momenta [5]

$$E^{(\nu,\pm1)}(1,2)_{\nu\to0} = \frac{\mathrm{i}}{2\pi q} \Big(\delta^2(k_1) - \delta^2(k_2) \Big),$$
$$E^{(\nu,\pm1)}(4,3)_{\nu\to0} = \frac{\mathrm{i}}{2\pi q} \Big(\delta^2(k_4) - \delta^2(k_3) \Big) . \quad (52)$$

Note that the second wave function has to be taken in conjugate form. Putting this into (51) and recalling (38) and the properties of the function G (39) and (40) we get

$$T = -\frac{1}{c(\nu_1,n_1)c(\nu_2,n_2)} \frac{4}{(2\pi)^2 q^2} G(q,0,-q)$$
$$= \frac{1}{c(\nu_1,n_1)c(\nu_2,n_2)} \frac{1}{\pi^2 q^2} \frac{1}{N_c} (H_2 P)(q) . \quad (53)$$

The last factor is just the BFKL Hamiltonian applied to the pomeron state.

As in Sect. 2, to find the pomeron $P(q)$ attached to the hadronic target we present it as the BFKL Green function applied to the color distribution in the hadron, (16). Again we need a mixed amputated–non-amputated Green function in the momentum space in its amputated part, (17). Applying the BFKL Hamiltonian we get

$$H_2 G(y,q,r) = \frac{1}{8\pi^2} qr \int \mathrm{d}\nu \omega(\nu,0) \mathrm{e}^{y\omega(\nu,0)} \frac{2^{-2\mathrm{i}\nu}(qr)^{2\mathrm{i}\nu}}{\nu^2+1/4} . \quad (54)$$

At large y with finite q and r we neglect all dependence on ν except in the exponential to obtain similarly to (19)

$$HG(y,q,r) = \frac{1}{2\pi^2} qr \Delta \mathrm{e}^{y\Delta} \sqrt{\frac{\pi}{ay}} . \quad (55)$$

Integration of r with the target color density converts it into the average target transverse radius R with a minus sign; see (16).

So we find for the matrix element T

$$T = -\frac{1}{2\pi^4 q^2} \frac{1}{c(\nu_1,n_1)c(\nu_2,n_2)} \frac{g_s^2}{N_c} qR\Delta \mathrm{e}^{y\Delta} \sqrt{\frac{\pi}{ay}} . \quad (56)$$

We have finally to do the integrations over ν_1 and ν_2 and summations over n_1 and n_2, which in the limit of high y_1 only take values ±1. These latter summations are trivial and give a factor 4. At $|n| = 1$ and small ν we have

$$\omega(\nu,\pm1) = -2\bar{\alpha}_s \zeta(3) \nu^2 . \quad (57)$$

One of the denominators in (46) reduces to ν^2 and is singular at $\nu \to 0$. However this singularity is cancelled by the square of the $1/c^2(\nu,\pm1)$ coming from (48) and (56). Therefore we find at small ν

$$\frac{\beta(\nu,\pm1)}{c^2(\nu,\pm1)} = 12\pi^3 \zeta(3) . \quad (58)$$

Neglecting all the rest ν-dependence except in the exponential in (45), integrations over ν_1 and ν_2 provide a factor

$$\frac{\pi}{2\bar{\alpha}_s \zeta(3) y_1} . \quad (59)$$

Combining all the factors we finally get for the cross section a simple expression:

$$\frac{\mathrm{d}\sigma^{(POO)}}{\mathrm{d}t} = 18\pi\xi N_c^6 F_c^{(POO)} \quad (60)$$
$$\times \int \mathrm{d}y\, g_s^6 \frac{\Delta}{N_c} \frac{b^2 R\zeta(3)}{q(q^2+M^2)^2} \mathrm{e}^{\Delta y} \frac{1}{\bar{\alpha}_s y_1} \sqrt{\frac{\pi}{ay}} .$$

It steadily grows as q^2 diminishes and behaves as $1/q$ at small q. Integrating over all q we find the cross section

$$\sigma^{(POO)} \quad (61)$$
$$= 9\pi^3 \xi N_c^6 F_c^{(POO)} \int \mathrm{d}y\, g_s^6 \frac{\Delta}{N_c} \frac{b^2 R\zeta(3)}{M^3} \mathrm{e}^{\Delta y} \frac{1}{\bar{\alpha}_s y_1} \sqrt{\frac{\pi}{ay}} .$$

It falls with Q^2 and the meson mass as $1/(Q^2 + m_{PS})^{3/2}$. The cross section has an order $\alpha_s(\alpha_s N_c)^6$, an order higher in $\alpha_s N_c$ than the leading contribution given by the reggeizing part (Fig. 2). This implies that the vertex POO has the same order $\alpha_s N_c$ as the triple pomeron vertex.

6 Numerical results

Both the cross section with a pure pomeron exchange and the one with a POO vertex have a simple dependence on the energies, which separates into a factor with the expected behavior at large rapidities. Separating also all the rest of the non-trivial factors we find the pure pomeron exchange contribution as follows:

$$\frac{\mathrm{d}\sigma^{(P)}}{\mathrm{d}t} = c^{(P)}\alpha_{\mathrm{em}}\alpha_{\mathrm{s}}(\bar{\alpha}_{\mathrm{s}})^5 b_0^2 \frac{m_{\eta_c}^2 R}{q^2 M^3} I\left(\frac{q}{M}\right) f^{(P)}(Y) , \quad (62)$$

where

$$f^{(P)}(Y) = \mathrm{e}^{\Delta Y}\sqrt{\frac{\pi}{aY}} \quad (63)$$

and

$$c^{(P)} = \frac{F_c^{(P)}}{5184\,\pi^6} . \quad (64)$$

The part originating from the POO vertex is

$$\frac{\mathrm{d}\sigma^{(POO)}}{\mathrm{d}t} \quad (65)$$

$$= c^{(POO)}\alpha_{\mathrm{em}}\alpha_{\mathrm{s}} \int \mathrm{d}y\, (\bar{\alpha}_{\mathrm{s}})^6 b_0^2 \frac{m_{\eta_c}^2 R}{q(q^2+M^2)^2} f^{(POO)}(Y,y) ,$$

where now

$$f^{(POO)}(Y,y) = \frac{1}{\bar{\alpha}_{\mathrm{s}}(Y-y)} \mathrm{e}^{\Delta y}\sqrt{\frac{\pi}{ay}} \quad (66)$$

and

$$c^{(POO)} = \frac{F_c^{(POO)}\zeta(3)\ln 2}{36\pi^4} . \quad (67)$$

The cross section with the POO vertex integrated over all transferred momenta is

$$\sigma^{(POO)} =$$

$$\frac{1}{2}\pi c^{(POO)}\alpha_{\mathrm{em}}\alpha_{\mathrm{s}} \int \mathrm{d}y\, (\bar{\alpha}_{\mathrm{s}})^6 b_0^2 \frac{m_{\eta_c}^2 R}{M^3} f^{(POO)}(Y,y) . \quad (68)$$

With $N_c = 3$ the color factors become

$$F_c^{(P)} = 200 \cdot 3^{-8}, \quad F_c^{(POO)} = 1600 \cdot 3^{-8} . \quad (69)$$

Regarding the region of integration in rapidity y, as explained in the introduction, we consider the interval $\delta Y < y < Y - \delta Y$ to warrant the use of the asymptotic forms for both the pomeron and the odderons. We choose $\delta Y = 3$.

One should also be cautious with the coupling constants in the expressions for the cross sections. In fact they refer to different scales relevant to the studied processes. Obviously one of the coupling constants refers to the coupling to the proton at a small and so non-perturbative scale. For the process mediated by the pure pomeron all other coupling constants are to be taken at the scale of the $\gamma^* \to \eta_c$ transition, that is the maximal of q and M. The cross section falls quite rapidly as q becomes larger than M, so that we can safely take M as the relevant scale. For the process with the POO transition however only three of the remaining six α refer to this scale. The other three are to be taken at an intermediate scale, characteristic for the POO transition at rapidity y. For high rapidities Y and y one can use the fact that the characteristic momenta k in the BFKL pomeron at rapidity Y have the order $\ln k \sim \sqrt{Y}$. Then one obtains a crude estimate for the coupling constant at the POO junction:

$$\alpha_{POO} \sim \sqrt{\frac{Y}{y}}\alpha_{\mathrm{s}}(M) . \quad (70)$$

We have taken $Y = \ln(1/x)$ with x defined as

$$x = \frac{m_{\eta_c}^2 + Q^2}{s + Q^2} . \quad (71)$$

Passing to concrete values of the coupling constants we take $\alpha_{\mathrm{s}}(M)$ as given by the leading order β function with three or four flavors N_f and $\Lambda_{\mathrm{QCD}} = 0.2\,\mathrm{GeV}/c$. The value of α_{s} at the POO junction was taken according to (70). As to the values of the pomeron intercept and its coupling to the proton, we have borrowed them from [11], where the proton structure function at small x was fitted by the pomeron exchange. From this fit one extracts both Δ and the product $\alpha_{\mathrm{s}}R$:

$$\Delta = 0.377, \quad \alpha_{\mathrm{s}}R = 0.096 \text{ fm} \quad (N_f = 3),$$
$$\alpha_{\mathrm{s}}R = 0.058 \text{ fm} \quad (N_f = 4) . \quad (72)$$

At first sight one may expect a large difference in the results for different N_f. However a smaller value of $\alpha_{\mathrm{s}}R$ for $N_f = 4$ is compensated for by a larger value of the rest of the coupling constants, so that the final results are practically independent of the number of flavors taken into account (see Fig. 7).

The calculation of the POO contribution (65) is straightforward. To find the contribution from the pure pomeron exchange (62) one has to calculate the integral (21). We did this using the standard Monte Carlo

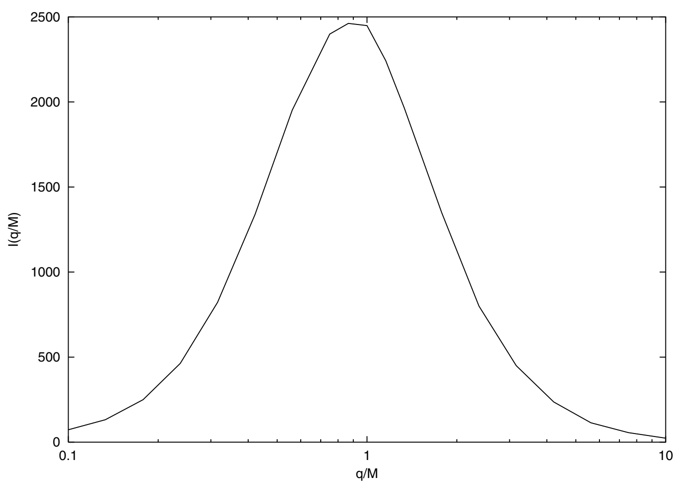

Fig. 4. The function $I(q/M)$ from (21)

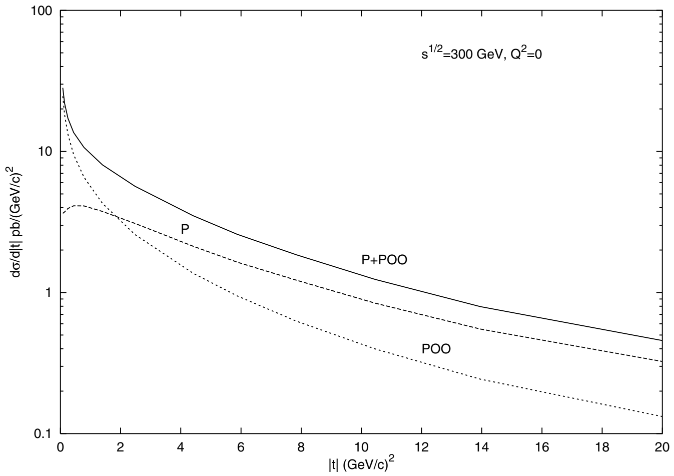

Fig. 5. Differential cross sections $\mathrm{d}\sigma/\mathrm{d}t$ from the reggeized pomeron exchange (P), POO transition (POO) and total at $Q^2 = 0$

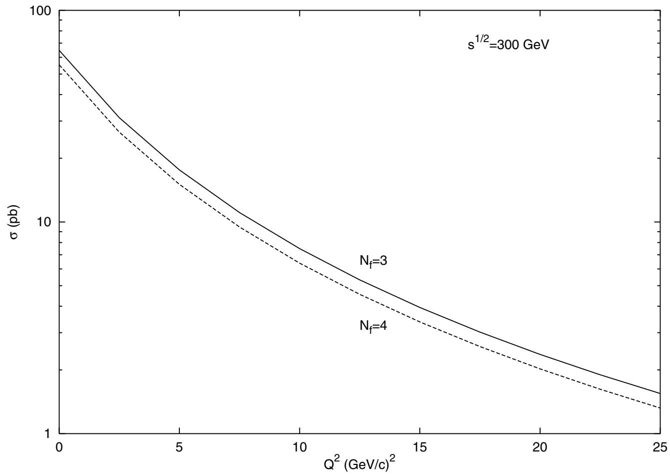

Fig. 7. Integrated cross sections σ for different number for flavors as a function of Q^2

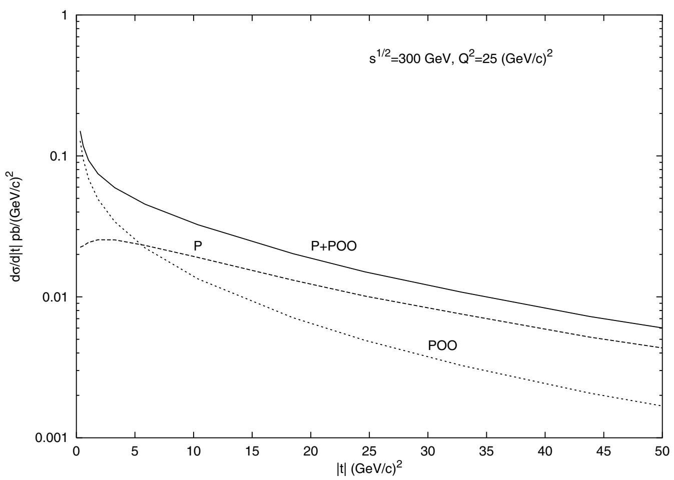

Fig. 6. Differential cross sections $\mathrm{d}\sigma/\mathrm{d}t$ from the reggeized pomeron exchange (P), POO transition (POO) and total at $Q^2 = 25\,(\mathrm{GeV}/c)^2$

Integrated over transferred momenta the total cross sections are found at $Q^2 = 0$ to be

$$\sigma^{(P)} = 34\,\mathrm{pb}, \quad \sigma^{(POO)} = 31\,\mathrm{pb},$$
$$\sigma = \sigma^{(P)} + \sigma^{(POO)} = 65\,\mathrm{pb}$$

and at $Q^2 = 25\,(\mathrm{GeV}/c)^2$

$$\sigma^{(P)} = 0.87\,\mathrm{pb}, \;\sigma^{(POO)} = 0.67\,\mathrm{pb},$$
$$\sigma = \sigma^{(P)} + \sigma^{(POO)} = 1.54\,\mathrm{pb}\,.$$

The total integrated cross sections σ (sum of P and POO) in the range of photon virtualities $0 < Q^2 < 25\,\mathrm{GeV}^{-2}$ are shown in Fig. 7. Here we present results for both $N_f = 3$ and 4. As one observes, the difference is quite insignificant.

With the growth of energy both contributions increase, preserving their shape in q. The increase is much more pronounced in the reggeizing pomeron contribution, since in this case the pomeron occupies the whole rapidity range, whereas for the POO transition this range is shorter.

program VEGAS. The results for $I(q/M)$ are presented in Fig. 4.

Our final results for the differential cross sections (62) and (65) and their sum (for $N_f = 3$) are presented in Figs. 5 and 6 for $\sqrt{s} = 300\,\mathrm{GeV}$ and $Q^2 = 0$ and $25\,(\mathrm{GeV}/c)^2$, respectively.

As we see, the contribution from the POO transition turns out to be of the same order as the one coming from the direct coupling of the pomeron to non-interacting gluons (P contribution).

However, the two contributions seem to behave differently at small transferred momenta. From (65) we see that at $q \to 0$ the POO contribution rises as $1/q$. On the other hand, the P contribution does not show such a behavior and seems to tend to a constant or zero at small q. This may help to see the POO contribution against the reggeizing pomeron contribution at very low q.

7 Conclusions

We have calculated the cross section of inclusive diffractive photo- and leptoproduction of η_c mesons, $\gamma^* p \to \eta_c + X$. We have considered the "triple Regge" contribution which contains the coupling POO of the pomeron to two odderons. The inclusion of a second contribution where the pomeron directly couples to two three-gluon states results in a significant increase of the cross section which grows with energy. However, in order to see the structure of the QCD odderon state with $C = -1$ state one has to select diffractive events with a large enough gap between the missing mass state "X" and the η_c. The total production rate is found to be of the order of 60 pb for photoproduction.

In our previous publication [5] we calculated the production rate for the quasielastic reaction $\gamma^* + p \to \eta_c + p$ due to odderon exchange. The photoproduction cross section was reported to be 27 pb, which would be smaller by a factor of 2 compared to the present case. However, note that for the quasielastic process considered in [5] one had to make the assumption of the non-perturbative odderon–proton coupling. In [5] we used the coupling proposed in [2], and we put the effective coupling constant α_s equal to unity. However, in a recent analysis of the pp and $p\bar{p}$ elastic scattering data this coupling constant has been estimated to be 0.3 [12]. With this value of the effective coupling constant the cross sections reported in [5] have to be reduced by a factor 30, and, in fact, the cross section of the inclusive η_c is much larger than the quasielastic one.

In the present calculation the very poorly known odderon-proton coupling does not enter. Instead one has to know also the non-perturbative pomeron–proton coupling, transformed into the value of the product $\alpha_s R$ where R is the effective proton radius. This product can be found with a much higher degree of reliability. In this study we have used the fit to the experimental proton structure function in [11]. Note that it gives physically reasonable values $\Delta = 0.377$ and $R = 0.59$ fm (for $N_f = 4$), which more or less agree with estimates made by different methods. Correspondingly, we feel that the cross sections found in this paper are much less affected by the uncertainty in the non-perturbative coupling of the proton.

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